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Beta-gamma directional correlation in the decay of Ag^{111}

By J. H. HAMILTON,[1] B.-G. PETTERSSON and J. M. HOLLANDER[2]

With 5 figures in the text

The directional correlation between the once-forbidden, nonunique 690 kev beta group and the 340 kev gamma ray in the decay of Ag^{111} has been measured as a function of beta energy from 153 to 595 kev. A magnetic lens spectrometer was used as the beta analyzer. The observed correlation is nearly isotropic. The anisotropy increases in a negative direction from ~ 0 at 150 kev to -0.023 ± 0.005 at 595 kev.

A comparison with theory indicates that cancellation among the nuclear matrix elements governing the beta decay is required to explain the small anisotropy. The energy dependence of the correlation indicates that second order terms should be included in the theoretical analysis. This is in agreement with the non-statistical shape observed for the 690 kev beta group.

Introduction

In a recent series of articles [1], Kotani and Ross have discussed the observables in once-forbidden beta decay. One problem which they discuss regards the possibility of determining the nuclear matrix elements (NME) in the nonunique, once-forbidden decay (1FNU).

They treat the once-forbidden beta decay transition-probability in terms of an expansion in powers of ϱ, the nuclear radius. In the first or "ξ approximation" one keeps only the first nonvanishing term for each observable in an expansion in powers of $\xi = \frac{\alpha Z}{2\varrho}$. The spectrum shape factor is a constant in the ξ-approximation. From the theoretical equations it follows that measurements which can be described entirely within the ξ-approximation can give information about the NME only in special cases. However, Kotani and Ross point out that when deviations from the ξ-approximation can be detected the situation is more promising. By measuring carefully several observables in a given decay and combining the results, it is in principle possible to obtain values of the NME.

In their Phys. Rev. Letter [1 a] Kotani and Ross mention Ag^{111} as one case where a careful study of the deviations from the ξ-approximation in several

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[1] National Science Foundation Postdoctoral Fellow on leave from Vanderbilt University.

[2] Guggenheim Fellow on leave from University of California.

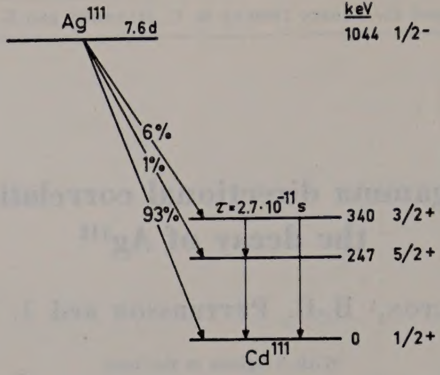

Fig. 1.

observables should yield useful information about NME. A nonstatistical spectrum shape has been reported for the 690 kev beta group in Ag^{111} by Robinson and Langer [2]. By including second order terms in the spectrum shape-factor, they fitted their data with the theoretical equation. From this they obtained values for the two parameters in the theoretical shape-factor which depend on combinations of the NME. However, further information is needed to obtain values for the NME themselves.

We have measured the directional correlation of the 690 kev beta group in coincidence with 340 kev gamma-ray in the hope of obtaining further information about the decay and the NME involved [3].

No previous measurements of beta-gamma directional correlations in Ag^{111} have been reported. From theoretical considerations and from the rather large deviation observed in the spectrum shape factor (17 % change in the shape factor in the range 80–600 kev^2), one would expect a reasonably large anisotropy for the 690 kev beta group − 340 kev gamma correlation. From theory, one expects the correlation coefficient $A_2 \sim \frac{1}{\xi}$. For Ag^{111} $\xi \approx 10$.

The life-time of the 340 kev level, 2.7×10^{-11} seconds [4], is short enough to make unlikely any significant attenuation of the correlation. This is not true for the 247 kev level, which has a life-time of 8.5×10^{-8} seconds [5]. The M1/E2 mixing ratio for the 340 kev gamma ray has been measured [6]. The decay-scheme and properties of the levels involved in the decay of Ag^{111} to Cd^{111} are well-known. The decay-scheme in Fig. 1 is taken from references 2 and 4. With all this information known about the decay, the only unknown quantities in the theoretical equations for the directional correlation of the 690 kev beta group − 340 kev gamma-ray are the NME.

Experimental procedures

Source preparation

Ag^{111} was prepared by neutron irradiation of natural palladium in the Kjeller pile, by means of the reaction Pd^{110} (n, γ) Pd^{111} $\xrightarrow{\beta^-}$ Ag^{111}. A silver fraction was

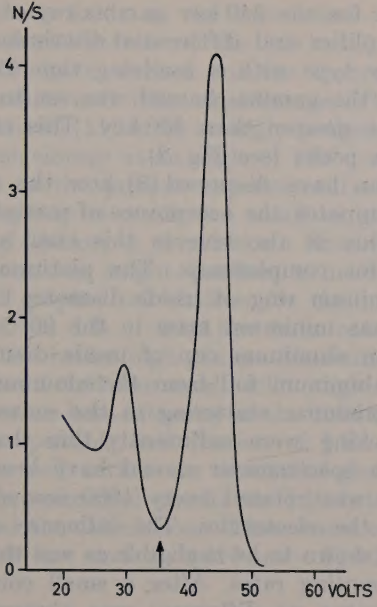

Fig. 2.

obtained essentially carrier-free by an anion-exchange column procedure in which the bulk of the palladium was initially adsorbed on a large Dowex A-1 column from 10 M HCl. At this HCl concentration, Ag^+ in trace concentrations is not appreciably adsorbed. The solution was then reduced in volume and passed through a small ($\sim 1 \times 8$ mm) column from 1 M HCl which retained the residual palladium and also the silver. This was followed by elution of Ag^+ with 10 M HCl. For added purification and a great reduction in volume, the last step was repeated with use of a third column of only a few microliters free volume. The Ag^{111} was eluted by two very small drops of 10 M HCl directly on to a platinum coated Zapon film with total thickness $\sim 200\ \mu$gm/cm^2. The diameter of the source was 5 mm and the thickness was estimated as $\approx 100\ \mu$g/cm^2. The gamma spectrum contained only the two wellknown Ag^{111} gamma-rays (see Fig. 2). An upper limit of 1 % was set for the relative intensity of any gamma-ray with energy very different from those of Ag^{111}. The source was checked for decay over a period of 3 half-lives and decayed with the reported [7] half-life of 7.6 days. Thus it was concluded that impurities, if present, were negligible.

Equipment

A magnetic lens spectrometer adapted for electron-gamma angular correlation experiments was used for these measurements. This instrument is described in a paper by Gerholm and Pettersson [8]. The momentum resolution was set a 8 % with a transmission of 3 %. The spectrometer was calibrated in terms of the K-line of the 661 kev gamma ray of Cs^{137}. A $2\frac{1}{2}'' \times 2''$ NaI (Tl) crystal optically coupled to a D.M. 6363 photomultiplier tube was used as the gamma detector.

The resolution was 12 % for the 340 kev gamma-ray of Ag^{111}. Each detector was followed by a linear amplifier and differential discriminator. The coincidence circuit was of the fast-slow type with a resolving time $2\tau_0 = 33\,m\mu\,s$.

The discriminator in the gamma-channel was set to accept all pulses which corresponded to energies greater than 300 kev. This energy is in the "valley" between the two gamma peaks (see Fig. 2).

Gerholm and Pettersson have discussed [8] how the construction of this spectrometer, in general, eliminates the acceptance of scattered gamma-radiation into the gamma detector. This is also true in this case, and details of the source mount are given here for completeness. The platinum coated Zapon foil was mounted on a thin aluminum ring of inside diameter 1.5 cm. This was placed in a lucite holder, which has minimum mass in the 90° and 270° directions. The lucite holder fits into an aluminum cup of inside diameter 5.5 cm. The source was grounded with an aluminum foil from the aluminum ring to the cup. This arrangement serves to minimize scattering in the source mounting. Furthermore the source and the backing were sufficiently thin that scattering of electrons from them into the beta spectrometer should have been negligible.

The gamma detector was rotated every 1000 seconds in order to avoid the effects of slow drifts in the electronics. The influence of the magnetic field on the gamma detector was shown to be negligible as was the influence of the gamma detector on the beta counting rates. After a small correction to normalize the total gamma counting rates, no difference was observed between the 90° and 270° data. No instrumental angular asymmetries have been observed in work with this spectrometer.

Treatment of data

The real coincidence rate was calculated from the equation $R = T - 2\tau_0 N_\gamma N_\beta$ where T is the total number of coincidences, N_γ and N_β the γ and β singles counting rates, $2\tau_0$ the resolving time. The gamma counting rates at each angle were measured to within 0.1 % every few hours. The small difference in the 90°–180° gamma singles counting rates, never greater than 0.8 % and generally much less, resulted from a slight misalignment of the gamma detector. N_γ and N_β were measured often as a check on the half-life and to aid in accurate calculations of the accidental coincidence rate.

The resolving time of the coincidence circuit was measured each night with good agreement in all cases. For these measurements the Ag^{111} source was used with a long delay time inserted into the circuit. The real-to-accidental ratio was >5 for every point. Errors in the correction for accidental coincidences were negligible. The delay curve of the coincidence circuit was checked each day and was adjusted to be in the middle of the flat region of maximum efficiency.

The long half-life of Ag^{111} and the procedure of rotating the gamma-detector made decay-corrections unnecessary.

The data have been corrected for the finite solid angles used [9]. With the geometry used the corrections were $A_\gamma = 0.96$ and $A_\beta = 0.92$ in the formula

$$A_2 \text{ corrected} = \frac{1}{A_\gamma} \cdot \frac{1}{A_\beta} \cdot A_2 \text{ measured.} \tag{1}$$

The effects of extraneous fields on the correlation should be quite small, if present, because of the short-life-time of the 340 kev level.

Several investigators [10] have worked out the theory of inner bremsstrahlung (I.B.) which compares rather well with experiments. The directional correlation of I.B. and beta-rays is known [11] to be strongly peaked in the forward direction, such that no I.B. photons emerge at an angle of 180° to the betas. This gives rise to a negative anisotropy in the present experimental setup. The intensity of the I.B. from the 690 kev β-group is much too small to give an observable contribution to the 690 kev β-340 kev γ correlation. However, I.B. from the more intense ground state transition might produce a measurable effect. From the theoretical equations and graphs, calculations for the number of coincidences in the 90° position arising from I.B. $-$1044 kev beta coicidences were made. The restrictions imposed by the rather high gamma-discriminator settings, the narrow beta-energy acceptance band and the small solid angle were stringent. Thus despite the factor of 10 greater beta-intensity of the 1044 kev group, an upper limit of 0.1 % was set for the possible contribution of I.B. to the 90° coincidences. This correction limit was well within the statistical deviation of the points and was not applied. Two experimental checks on I.B.-beta coincidences were also made, and these confirmed the calculations. These cases corresponded to accepting $E_\gamma \geqslant 100$ kev $- E_\beta = 760 \pm 60$ kev and $E_\gamma \geqslant 410$ kev $- E_\beta = 250 \pm 30$ kev.

Results

The directional correlation between the 690 kev beta-group and the 340 kev gamma ray was measured at five beta-energies in the range 153 to 595 kev. Data were taken at 90°, 180° and 270°, and the 90° and 270° data were combined. Over 200,000 real coincidences were taken at both 90° + 270° and 180° at each of the four lowest beta-energies and over 120,000 at the highest beta-energy. Each of the five points took from 30 to 80 hours running time, which covered a period from 2 to 7 days.

The directional correlation function is

$$N(\theta) = 1 + A_2 P_2 (\cos\theta) \tag{2}$$

$$= 1 + A_2 \left(\tfrac{3}{2}\cos^2\theta - \tfrac{1}{2}\right) \tag{3}$$

and the coefficient A_2 is given by

$$\varepsilon = A_2 = \frac{2A}{3+A}, \tag{4}$$

where A is the measured anisotropy,

$$A = \frac{N(\pi) - N(\pi/2)}{N(\pi/2)}. \tag{5}$$

In Fig. 3 we show our results in the form of a plot of A_2 vs. W, where W is the relativistic electron energy. The A_2 values are corrected for all the effects mentioned in the section on treatment of the data. The errors given in the figure

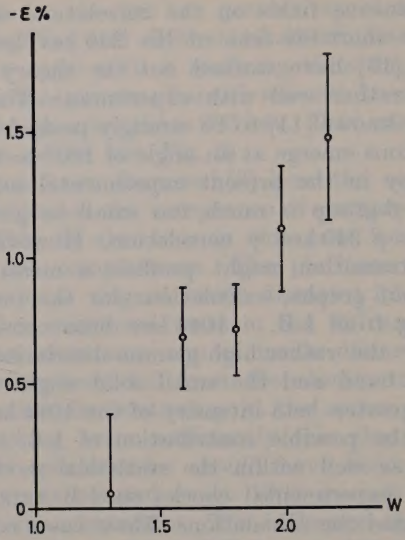

Fig. 3.

correspond to one standard deviation. A_2 increases in a negative direction with increasing beta-energy. The maximum measured value of A_2 is -0.015 ± 0.003 at a beta energy of 595 kev.

Theoretical interpretation

The theory of β–γ angular correlations in once forbidden, nonunique (1FNU) beta decay has been treated extensively by several authors [1, 12]. In our theoretical analysis, we make use of the work of Kotani and Ross, who have discussed the general question of nuclear matrix elements in 1FNU beta decay. In the "ξ approximation" of Kotani and Ross, only the leading nonvanishing term for any observable is kept in the expansion of the transition probability in descending powers of $\xi \equiv \alpha Z/2\varrho \approx Z/A^{\frac{1}{3}}$. This gives the well known energy-independent shape correction factor for the beta spectrum.

$$C(W) = |V|^2 + |Y|^2 \tag{6}$$

where V and Y are functions of the NME [1c].

The β–γ directional correlation coefficient A_2 in the ξ approximation is given by

$$A_2 = \lambda_2 \frac{p^2}{W} \cdot \frac{R_3}{C(W)} \tag{7}$$

if time reversal invariance is assumed. p and W are the relativistic momentum and energy and $C(W)$ is given by Eq. (6). λ_2 contains coulomb corrections of order $(\alpha Z W/p)$, which are available from the tables and graphs of Kotani and Ross [1c]. If these are neglected, $\lambda_2 = 1$. R_3 is energy independent and depends on the NME and other parameters like the gamma ray multipole mixing ratio

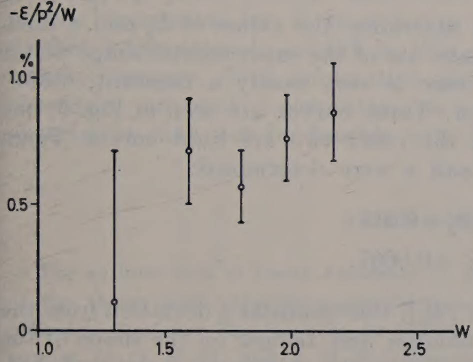

Fig. 4.

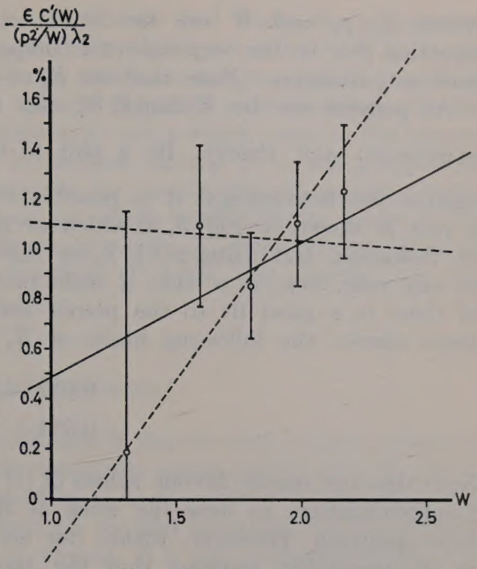

Fig. 5.

and nuclear spins (see 1 c for details). One expects $R_3/C(W) \sim \frac{1}{\xi} \approx 0.1$. Fig. 4 gives a plot of $\varepsilon W/p^2$ vs. W. Note $R_3/C(W) \approx 0.01 \approx \frac{1}{\xi^2}$. Since Eq. (7) neglected terms of order $\frac{1}{\xi^2}$, it may be expected that Eq. (7) is not sufficient to describe this case.

When the next terms in the ξ-expansion are retained, the shape factor is no longer energy-independent, but has the form

$$C(W) = k\left(1 + a\,W + \frac{b}{W} + c\,W^2\right) = k\,C'(W). \tag{8}$$

k, a, b, and c depend on combinations of the NME [1 b, c].

The criterion for applicability of the ξ-approximation, that $\xi \gg W_0$, is in this case well-fulfilled ($\xi \approx 10$, $W_0 = 2.3$) so it would have been expected that the ξ-approximation should describe the data to within $1/\xi \sim 10\,\%$. However, Robinson and Langer [2] have observed a deviation from linearity of 17 % in the Fermi plot of the 690 kev beta group, which indicates a definite failure of the ξ-approximation. They have found that the observed shape of the Ag^{111} spectrum can be fitted with the shape factor $C'(W) = 1 + 0.55/W$; thus, in equation (8) the constants are $a \approx 0$, $b = 0.55$, and $c = 0$ (c is generally approximately zero).

Similarly, in the β–γ directional correlation, the addition of the next higher term in the ξ-expansion results in the expression

$$A_2 = \lambda_2\,\frac{p^2}{W}\,\frac{(R_3 + e\,W)}{C'(W)} \tag{9}$$

where λ_2, p, and W are the same as in equation (7), and $C'(W)$, defined in equation (8), is the experimental shape factor. e is a new energy-independent nuclear parameter. (Note that the R_3 of equation (9) equals R_3/k of equation (7).)

As pointed out by Kotani [1 d], this equation is convenient for comparison of experiment and theory. By a plot of the energy-dependent quantity $\frac{\varepsilon C'(W)}{(p^2/W)\lambda_2}$ against the beta-energy, it is possible to determine the values of R_3 and e. Such a plot is shown in Fig. 5, in which we make use of the experimental shape factor of Robinson and Langer [2]. λ_2 in this case is very nearly a constant, 0.88—in any case, here it is only a scale factor. Three curves are seen in Fig. 5; one of these is a good fit to the points and the other two are limit curves. From these curves, the following limits of R_3 and e were determined:

$$-0.012 \leqslant R_3 \leqslant 0.016$$

$$-0.014 \leqslant e \leqslant 0.0007.$$

Note that our results favour values of $|e| \gtrsim |R_3|$; this indicates a deviation from the ξ-approximation, as does the work of Robinson and Langer on the shape of the beta spectrum. However, within our error limits, the value $e \approx 0$ is not excluded, so the possibility remains that the ξ-approximation is able to account for the β–γ directional correlation.

In view of the rather large deviation of the spectrum from the allowed shape (failure of the ξ-approximation), the small maximum β–γ anisotropy observed for Ag^{111} is somewhat surprising. In search of an explanation for this small value, one notes first the sensitivity of the correlation function to the $M1-E2$ mixing ratio of the 340 kev transition. The correlation coefficient parameters R_3 and e for this decay are given by Kotani and Ross [1 e] in the Konopinski–Uhlenbeck approximation, where $(\alpha Z)^2$ terms ($C-5$ of 1 b), finite size effects ($3-3$a of 1 b) and third forbidden contributions ($C-1$ of 1 b) are neglected, as:

$$R_3 k = \left[\frac{1}{6}(2x-u) - \frac{1}{2\sqrt{10}}z\right]\left[Y - \frac{1}{3}(x-u)W_0\right]\Delta_1 \qquad (10)$$

$$e k = -\frac{1}{72}\left[(2x+7u)(2x-u) - \frac{6}{\sqrt{10}}(5u-2x)z + \frac{21}{10}z^2\right]\Delta_1. \qquad (11)$$

Here x, u, z, and Y are nuclear matrix element parameters as defined by Kotani [1 d][1], and

$$\Delta_1 = (1 + 2\sqrt{3}\,\delta - \delta^2)(1+\delta^2)^{-1}. \qquad (12)$$

δ is the ratio of $E2$ and $M1$ reduced photon transition probabilities.

$$\delta = \frac{(j_1 \| L_1 + 1 \| j_2)}{(j_1 \| L_1 \| j_2)}. \qquad (13)$$

It follows from equation (12) that Δ_1 vanishes for $\delta = -0.27$. From Coulomb excitation measurements on Cd^{111} McGowan and Stelson obtained [6] $\delta = -0.39 \pm$

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[1] Ref 1 d eq. A 5 gives due to a misprint the second factor in our eq. 10 as $Y - \frac{1}{3}(z-u)W_0$.

± 0.02. (This is expressed in the notation of Kotani and Ross; the notation of McGowan and Stelson is opposite in sign.) With this latter value of δ, Δ_1 is calculated to be -0.44. This in itself is not small enough to explain the small observed value of the anisotropy (small value of R_3). One must therefore consider cancellations among the nuclear parameters x, u, and z which occur in the expression for R_3. Since another cancellation is required among x, u, and y appearing in the expression for Y in order to explain the non-statistical spectral shape, further information is needed in this case to obtain more definite limits on the matrix elements. Such information might be given by a measurement of the β-circularly polarized γ correlation as a function of angle and energy, but this experiment may not be feasible with Ag^{111}.

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Institute of Physics, University of Uppsala, Uppsala, Sweden.

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